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2009 J. Phys. A: Math. Theor. 42 115303

(http://iopscience.iop.org/1751-8121/42/11/115303)

IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **42** (2009) 115303 (8pp) doi:10.1088/1751-8113/42/11/115303

Quantum teleportation and state sharing using a genuinely entangled six-qubit state

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Received 3 January 2009, in final form 5 February 2009
Published 23 February 2009
Online at stacks.iop.org/JPhysA/42/115303

Abstract
The usefulness of the genuinely entangled six-qubit state that has recently been introduced by Borras *et al* (2007 *J. Phys. A: Math. Theor.* **40** 13407) is investigated for the quantum teleportation of an arbitrary three-qubit state and for quantum state sharing (QSTS) of an arbitrary two-qubit state. We construct two distinct protocols for QSTS of an arbitrary two-qubit state using this state as an entangled channel. We construct 16 orthogonal four-qubit states which can lock an arbitrary two-qubit state between two parties.

PACS numbers: 03.67.Hk, 03.65.Ud

1. Introduction

Entanglement is the most striking and counter-intuitive feature of quantum mechanics that has found many practical applications in the field of cryptography and communication technology [1]. Entangled states such as the Bell, GHZ and their generalizations play a significant role in the accomplishment of various quantum tasks such as teleportation [2], secret sharing [3] and dense coding [4]. It is well characterized only up to four qubits [5]. Intriguingly, not all entangled states are useful in carrying out the desired operations.

In the case of three qubits, entanglement can be characterized into two inequivalent ways [6]: the GHZ and W state categories. While the GHZ states are suitable for carrying out various quantum tasks, the normal W states [7] are not. As is evident, the nature of the multipartite entanglement is crucial in determining the efficacy of the entangled state under consideration for quantum communication. The GHZ states have long-range order characteristically different from the W state, for which such order is absent, although it has greater local connectivity.

Bennett *et al* [2] introduced the first scheme for the teleportation of an arbitrary single-qubit state $|\psi_a\rangle = \alpha|0\rangle + \beta|1\rangle$, where $\alpha, \beta \in C$ and $|\alpha|^2 + |\beta|^2 = 1$ using an EPR pair as an entangled resource. This has been experimentally achieved both in laboratory as well as in realistic conditions [8, 9]. Recently, attention has turned toward the investigation of the efficacy of a number of multipartite entangled channels for the teleportation of an arbitrary two-qubit state given by [10–13]

$$|\psi_b\rangle = \alpha|00\rangle + \mu|10\rangle + \gamma|01\rangle + \beta|11\rangle, \tag{1}$$

where $\alpha, \mu, \gamma, \beta \in C$ and $|\alpha|^2 + |\mu|^2 + |\gamma|^2 + |\beta|^2 = 1$. Further, it was shown by two of the present authors that it is possible to teleport an arbitrary N qubit state of type

$$|\psi_N\rangle = \sum_{i_1,i_2,\ldots,i_n=0}^{1} \alpha_{i_1 i_2 \cdots i_n} |i_1 i_2 \cdots i_n\rangle, \tag{2}$$

where $\alpha_{i_1 i_2 \cdots i_n} \in C$ and $\Sigma|\alpha_{i_1 i_2 \ldots i_n}|^2 = 1$, using a $2N$ qubit state of the form [14]:

$$|\zeta_{2N}\rangle = \sum_{i_1,i_2,\ldots,i_n=0 i_1=i_2\cdots=i_n\neq 1}^{1} R(i_1 i_2 \cdots i_n)|i_1 i_2 \cdots i_n\rangle - |1\rangle^{\otimes n}. \tag{3}$$

Here, R refers to the unitary 'reflection operator' performing the transformation $|i_1, i_2, \ldots, i_n\rangle \to |i_n, \ldots, 1_2, i_1\rangle$.

Quantum state sharing (QSTS) [15] is the use of controlled teleportation for the secret sharing of quantum information among various parties such that the receiver can obtain the required information, only if all the members involved in the protocol cooperate. Hillery *et al* [16] proposed the first scheme for the QSTS of a single-qubit state $|\psi_a\rangle$ using a tri-partite GHZ state. Later, the usefulness of an asymmetric W state [17] was demonstrated for the same purpose and was experimentally realized in ion trap systems.

QSTS of a two-qubit state $|\psi_b\rangle$ was initially realized using four Bell pairs [18]. Recently, two of the present authors have proposed that QSTS of $|\psi_b\rangle$ can be realized using the highly entangled five-partite states which are not decomposable into Bell pairs of type [12],

$$|\psi_5\rangle = \tfrac{1}{2}(|\Omega_1\rangle|\phi_-\rangle + |\Omega_2\rangle|\psi_-\rangle + |\Omega_3\rangle|\phi_+\rangle + |\Omega_4\rangle|\psi_+\rangle), \tag{4}$$

where $|\Omega_i\rangle$ form a tri-partite orthogonal basis. The same has been achieved by using the cluster state [19]

$$|C_N\rangle = \frac{1}{2^{N/2}} \otimes_{a=1}^{N} \left(|0\rangle_a \sigma_z^{a+1} + |1\rangle_a\right), \tag{5}$$

with $\sigma_z^{N+1} = 1$. In the experimental realization of multi-partite entangled states [20], one often starts with multiple copies of the Bell states which are subsequently further entangled. In an analogous manner, the theoretical search for multi-partite entangled states often takes recourse to the assembling of the desired state from constituents of Bell, GHZ states, etc. It is worth observing that the construction of higher dimensional states relies on computational optimization schemes [21] and may not be familiar from physical considerations. Therefore, the efficacy of these states needs to be checked with several quantum protocols.

Borras *et al* [22] introduced a genuinely entangled six-qubit state which is not decomposable into pairs of Bell states. It is given by

$$\begin{aligned}|\psi_6\rangle = \tfrac{1}{4}[&|000\rangle(|0\rangle|\phi_+\rangle + |1\rangle|\psi_+\rangle) + |001\rangle(|0\rangle|\psi_-\rangle - |1\rangle|\phi_-\rangle)\\ &+ |010\rangle(|0\rangle|\psi_+\rangle - |1\rangle|\phi_+\rangle) + |011\rangle(|0\rangle|\phi_-\rangle + |1\rangle|\psi_-\rangle)\\ &+ |100\rangle(-|0\rangle|\psi_-\rangle - |1\rangle|\phi_-\rangle) + |101\rangle(-|0\rangle|\phi_+\rangle + |1\rangle|\psi_+\rangle)\\ &+ |110\rangle(|0\rangle|\phi_-\rangle - |1\rangle|\psi_-\rangle) + |111\rangle(|0\rangle|\psi_+\rangle + |1\rangle|\phi_+\rangle)].\end{aligned} \tag{6}$$

Here $|\psi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ and $|\phi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$ refer to the Bell states. As is understood, six-partite entangled states are rather difficult to create. To the best of the authors' knowledge, only one kind of six-partite states, i.e., the cluster states which consist of four superposition terms, have been created in laboratory conditions [20]. The difficulty in creating entangled states increases with the increase in the number of superposition terms and is a challenge for experimentalists. However, states with a large number of superposition terms are in general more advantageous than states with fewer terms. For instance, the cluster state cannot be used for teleporting a three-qubit state but $|\psi_6\rangle$, which is different from the GHZ and the cluster states under LOCC, can be used for this purpose. This state could be realized by implementing the appropriate CNOT and Hadamard gates initially to three Bell pairs. A detailed investigation into the experimental creation of this state is underway and could enhance the applicability of this state.

This state exhibits genuine entanglement according to many measures. The reduced single-, two- and three-qubit density matrices of this state are all completely mixed. Further, it has been pointed out that no other pure state of six qubits has been found that evolves to a mixed state with a higher amount of entanglement [23]. This state also satisfies the monogamy inequality given by [24],

$$\sum_{i=2}^{n} C^2_{A_1 A_i} \leqslant C^2_{A_1|A_2\cdots A_n}, \tag{7}$$

where $C_{A|B}$ represents the concurrence between the subsystems A and B. Hence, this state turns out to be an important resource for quantum communication protocols. Here, we show that this state can be used for the teleportation of an arbitrary three-qubit state and for the QSTS of an arbitrary two-qubit state in two distinct ways. Six-qubit cluster state entanglement has been realized in laboratory conditions. As it is true for the GHZ and the cluster state, it is always possible for one to project two qubits into the Bell state by performing local measurements on the other qubits in $|\psi_6\rangle$. Hence, $|\psi_6\rangle$ is 'maximally connected' and can be used for the teleportation of an arbitrary single-qubit state.

It is also difficult to disentangle this state by performing local operations, and entanglement still prevails after three local measurements making the state 'persistent' than the GHZ states. The decoherence properties of this state have been well studied in the literature. It has been shown that entanglement of $|\psi_6\rangle$ decays more slowly than that of the GHZ state and is similar to the W state. Further, it has been shown that the entanglement of $|\psi_6\rangle$ is robust against the depolarizing channel. These factors give us motivation to investigate this six-qubit entangled channel for the above-mentioned quantum communication purposes.

2. Teleportation

Let Alice and Bob have the first three and the last three qubits in $|\psi_6\rangle$, respectively. Alice has an arbitrary three-qubit state given by

$$|\psi_3\rangle = \sum_{i_1,i_2,i_3=0}^{1} \alpha_{i_1 i_2 i_3} |i_1 i_2 i_3\rangle, \tag{8}$$

where $\alpha_{i_1 i_2 i_3} \in C$ and $\Sigma|\alpha_{i_1 i_2 i_3}|^2 = 1$ which she wants to teleport to Bob. Now Alice can perform a six-qubit measurement on her system of qubits and convey the outcome of her measurement to Bob via six cbits. For instance, if the outcome of Alice's measurement is

$$\frac{1}{\sqrt{8}} \sum_{i_1,i_2,\ldots,i_3=0}^{1} |i_1 i_2 i_3\rangle |i_1 i_2 i_3\rangle, \tag{9}$$

Table 1. The outcome of the measurement performed by Alice and the state obtained by Bob and Charlie (the coefficient $\frac{1}{2}$ is removed for convenience).

Outcome of the measurement	State obtained
$\|0000\rangle + \|1001\rangle \pm \|0111\rangle \pm \|1110\rangle$	$\alpha\|\eta_1\rangle + \mu\|\eta_2\rangle \pm \gamma\|\eta_3\rangle \pm \beta\|\eta_4\rangle$
$\|0000\rangle - \|1001\rangle \pm \|0111\rangle \mp \|1110\rangle$	$\alpha\|\eta_1\rangle - \mu\|\eta_2\rangle \pm \gamma\|\eta_3\rangle \mp \beta\|\eta_4\rangle$
$\|0001\rangle + \|1000\rangle \pm \|0110\rangle \pm \|1111\rangle$	$\alpha\|\eta_2\rangle + \mu\|\eta_1\rangle \pm \gamma\|\eta_4\rangle \pm \beta\|\eta_3\rangle$
$\|0001\rangle - \|1000\rangle \pm \|0110\rangle \mp \|1111\rangle$	$\alpha\|\eta_2\rangle - \mu\|\eta_1\rangle \pm \gamma\|\eta_4\rangle \mp \beta\|\eta_3\rangle$
$\|0011\rangle + \|1010\rangle \pm \|0100\rangle \pm \|1101\rangle$	$\alpha\|\eta_3\rangle + \mu\|\eta_4\rangle \pm \gamma\|\eta_1\rangle \pm \beta\|\eta_2\rangle$
$\|0011\rangle - \|1010\rangle \pm \|0100\rangle \mp \|1101\rangle$	$\alpha\|\eta_3\rangle - \mu\|\eta_4\rangle \pm \gamma\|\eta_1\rangle \mp \beta\|\eta_2\rangle$
$\|0010\rangle + \|1011\rangle \pm \|0101\rangle \pm \|1100\rangle$	$\alpha\|\eta_4\rangle + \mu\|\eta_3\rangle \pm \gamma\|\eta_2\rangle \pm \beta\|\eta_1\rangle$
$\|0010\rangle - \|1011\rangle \pm \|0101\rangle \mp \|1100\rangle$	$\alpha\|\eta_4\rangle - \mu\|\eta_3\rangle \pm \gamma\|\eta_2\rangle \mp \beta\|\eta_1\rangle$

then Bob's system collapses to $\sum \alpha_{i_1,i_2,i_3}|\zeta_{i_1 i_2 1_3}\rangle$, where $|\zeta_{i_1 i_2 1_3}\rangle$ are given by

$$|\zeta_{000}\rangle = (|0\rangle|\phi_+\rangle + |1\rangle|\psi_+\rangle), \tag{10}$$

$$|\zeta_{001}\rangle = (|0\rangle|\psi_-\rangle - |1\rangle|\phi_-\rangle), \tag{11}$$

$$|\zeta_{010}\rangle = (|0\rangle|\psi_+\rangle - |1\rangle|\phi_+\rangle), \tag{12}$$

$$|\zeta_{011}\rangle = (|0\rangle|\phi_-\rangle + |1\rangle|\psi_-\rangle), \tag{13}$$

$$|\zeta_{100}\rangle = (-|0\rangle|\psi_-\rangle - |1\rangle|\phi_-\rangle), \tag{14}$$

$$|\zeta_{101}\rangle = (-|0\rangle|\phi_+\rangle + |1\rangle|\psi_+\rangle), \tag{15}$$

$$|\zeta_{110}\rangle = (|0\rangle|\phi_-\rangle - |1\rangle|\psi_-\rangle), \tag{16}$$

$$|\zeta_{111}\rangle = (|0\rangle|\psi_+\rangle + |1\rangle|\phi_+\rangle). \tag{17}$$

Now, Bob can perform an appropriate unitary operation on his qubits and obtain $|\psi_3\rangle$. Each measurement could be further broken down into simpler parts using Hadamard and computational basis which might render this scheme to be experimentally feasible once $|\psi_6\rangle$ has been created. For instance, the measurement outcome in equation (9) could be rewritten as

$$\sum_{i=1}^{4} \Omega_i(|0\rangle + |1\rangle)\Omega_i(|0\rangle + |1\rangle) + \sum \Omega_i(|0\rangle - |1\rangle)\Omega_i(|0\rangle - |1\rangle), \tag{18}$$

where $\Omega_i \in (|00\rangle, |01\rangle, |10\rangle, |11\rangle)$ for $i = 1, 2, 3, 4$, respectively. We shall investigate the usefulness of this state for QSTS of an arbitrary two-qubit state in the following sections.

3. QSTS of an arbitrary two-qubit state

3.1. Protocol 1

We let Alice possess particles 1 and 2, Bob possess particles 3 and 4 and Charlie possess particles 5 and 6 in $|\psi_6\rangle$, respectively. Alice also has $|\psi_b\rangle$ which she wants to lock between Bob and Charlie. To achieve this, Alice performs a von-Neumann joint four-particle measurement on her qubits and conveys the outcome of her measurement to Charlie by four cbits of information. The outcome of the joint measurement made by Alice and the entangled state obtained by Bob and Charlie are shown in table 1. Here $|\eta_i\rangle$ are given by

Table 2. The outcome of the measurement performed by Bob and the state obtained by Charlie.

Outcome of the measurement	State obtained
$\|\phi_+\rangle$	$\alpha\|11\rangle - \gamma\|00\rangle + \beta\|10\rangle + \mu\|01\rangle$
$\|\phi_-\rangle$	$\alpha\|00\rangle - \gamma\|11\rangle - \beta\|01\rangle + \mu\|10\rangle$
$\|\psi_+\rangle$	$\alpha\|01\rangle + \gamma\|10\rangle - \beta\|00\rangle - \mu\|11\rangle$
$\|\psi_-\rangle$	$\alpha\|10\rangle - \gamma\|01\rangle + \beta\|11\rangle - \mu\|00\rangle$

$$|\eta_1\rangle = \tfrac{1}{2}(|\phi_-\rangle|00\rangle + |\phi_+\rangle|11\rangle + |\psi_+\rangle|01\rangle + |\psi_-\rangle|10\rangle), \tag{19}$$

$$|\eta_2\rangle = \tfrac{1}{2}(-|\psi_-\rangle|00\rangle - |\psi_+\rangle|11\rangle + |\phi_+\rangle|01\rangle + |\phi_-\rangle|10\rangle), \tag{20}$$

$$|\eta_3\rangle = \tfrac{1}{2}(|\phi_+\rangle|00\rangle - |\phi_-\rangle|11\rangle - |\psi_-\rangle|01\rangle + |\psi_+\rangle|10\rangle), \tag{21}$$

$$|\eta_4\rangle = \tfrac{1}{2}(-|\psi_+\rangle|00\rangle + |\psi_-\rangle|11\rangle + |\phi_+\rangle|10\rangle - |\phi_-\rangle|01\rangle). \tag{22}$$

It could be noted that the four-partite measurement outcomes in table 1 could be further broken down into Bell and single-partite measurements . For instance, the first measurement could be further broken down as

$$(|\psi_+\rangle(|0\rangle + |1\rangle) + |\psi_-\rangle(|0\rangle - |1\rangle))|0\rangle + (|\phi_-\rangle(|0\rangle - |1\rangle) + |\phi_+\rangle(|0\rangle + |1\rangle))|1\rangle, \tag{23}$$

where $|\psi_+\rangle$, $|\psi_-\rangle$, $|\phi_+\rangle$ and $|\phi_-\rangle$ refer to the Bell states, respectively. Bob can perform a two-qubit measurement on his qubits and communicate the outcome of his measurement to Charlie who then performs an appropriate unitary transformation to get the state $|\psi_b\rangle$. Suppose the Bob–Charlie system collapses to $\alpha|\eta_1\rangle + \mu|\eta_2\rangle + \gamma|\eta_3\rangle + \beta|\eta_4\rangle$, then if Bob wants to perform a Bell measurement, the outcome of the measurement performed by Bob and the state received by Charlie are shown in table 2. Charlie can apply a suitable unitary operator on his qubits to get back the state $|\psi_b\rangle$. The state obtained by Charlie and the corresponding unitary operator applied are shown in table 3. Since the scheme involves only Bell and Hadamard measurements, this scheme might be experimentally feasible.

3.2. Protocol II

We can also demonstrate a different protocol for the QSTS of $|\psi_b\rangle$ using $|\psi_6\rangle$ as an entangled resource by redistributing the particles among Alice, Bob and Charlie. In this protocol, we let Alice possess particles 1, 2 and 3; Bob possess particle 4 and Charlie possess particles 5 and 6 in $|\psi_6\rangle$, respectively. In order to teleport $|\psi_b\rangle$, Alice performs a joint five-particle measurement on her qubits and conveys the outcome of her measurement to Charlie by four cbits of information. The measurement performed by Alice and the corresponding entangled states obtained by Bob and Charlie are shown in table 4.

Here $|\zeta_i\rangle$ are given by

$$|\zeta_1\rangle = \tfrac{1}{\sqrt{2}}(|0\rangle|\phi_+\rangle + |1\rangle|\psi_+\rangle), \tag{24}$$

$$|\zeta_2\rangle = \tfrac{1}{\sqrt{2}}(|0\rangle|\psi_-\rangle - |1\rangle|\phi_-\rangle), \tag{25}$$

$$|\zeta_3\rangle = \tfrac{1}{\sqrt{2}}(|0\rangle|\psi_+\rangle - |1\rangle|\phi_+\rangle), \tag{26}$$

$$|\zeta_4\rangle = \tfrac{1}{\sqrt{2}}(|0\rangle|\phi_-\rangle + |1\rangle|\psi_-\rangle), \tag{27}$$

$$|\zeta_5\rangle = \tfrac{1}{\sqrt{2}}(-|0\rangle|\psi_-\rangle - |1\rangle|\phi_-\rangle), \tag{28}$$

Table 3. Set of unitary operators required to obtain $|\psi\rangle_b$.

State	Unitary operation
$(\alpha\|01\rangle + \gamma\|00\rangle + \mu\|11\rangle + \beta\|10\rangle)$	$I \otimes \sigma_1$
$(\alpha\|01\rangle + \gamma\|00\rangle - \mu\|11\rangle - \beta\|10\rangle)$	$\sigma_3 \otimes \sigma_1$
$(\alpha\|01\rangle - \gamma\|00\rangle + \mu\|11\rangle - \beta\|10\rangle)$	$I \otimes \mathrm{i}\sigma_2$
$(\alpha\|01\rangle - \gamma\|00\rangle - \mu\|11\rangle + \beta\|10\rangle)$	$\sigma_3 \otimes \mathrm{i}\sigma_2$
$(\alpha\|11\rangle + \gamma\|10\rangle + \mu\|01\rangle + \beta\|00\rangle)$	$\sigma_1 \otimes \sigma_1$
$(\alpha\|11\rangle - \gamma\|10\rangle + \mu\|01\rangle - \beta\|00\rangle)$	$\sigma_1 \otimes \mathrm{i}\sigma_2$
$(\alpha\|11\rangle + \gamma\|10\rangle - \mu\|01\rangle - \beta\|00\rangle)$	$\mathrm{i}\sigma_2 \otimes \sigma_1$
$(\alpha\|11\rangle - \gamma\|10\rangle - \mu\|01\rangle + \beta\|00\rangle)$	$\mathrm{i}\sigma_2 \otimes \mathrm{i}\sigma_2$
$(\alpha\|00\rangle + \gamma\|01\rangle + \mu\|10\rangle + \beta\|11\rangle)$	$I \otimes I$
$(\alpha\|00\rangle - \gamma\|01\rangle + \mu\|10\rangle - \beta\|11\rangle)$	$I \otimes \sigma_3$
$(\alpha\|00\rangle + \gamma\|01\rangle - \mu\|10\rangle - \beta\|11\rangle)$	$\sigma_3 \otimes I$
$(\alpha\|00\rangle - \gamma\|01\rangle - \mu\|10\rangle + \beta\|11\rangle)$	$\sigma_3 \otimes \sigma_3$
$(\alpha\|10\rangle + \gamma\|11\rangle + \mu\|00\rangle + \beta\|01\rangle)$	$\sigma_1 \otimes I$
$(\alpha\|10\rangle - \gamma\|11\rangle + \mu\|00\rangle - \beta\|01\rangle)$	$\sigma_1 \otimes \sigma_3$
$(\alpha\|10\rangle + \gamma\|11\rangle - \mu\|00\rangle - \beta\|01\rangle)$	$\mathrm{i}\sigma_2 \otimes I$
$(\alpha\|10\rangle - \gamma\|11\rangle - \mu\|00\rangle + \beta\|01\rangle)$	$\mathrm{i}\sigma_2 \otimes \sigma_3$

Table 4. The outcome of the measurement performed by Alice and the state obtained by Bob and Charlie (the coefficient $\frac{1}{2}$ is removed for convenience).

Outcome of the measurement	State obtained
$\|00000\rangle + \|10001\rangle \pm \|01011\rangle \pm \|11010\rangle$	$\alpha\|\zeta_1\rangle + \mu\|\zeta_2\rangle \pm \gamma\|\zeta_4\rangle \pm \beta\|\zeta_3\rangle$
$\|00000\rangle - \|10001\rangle \pm \|01011\rangle \mp \|11010\rangle$	$\alpha\|\zeta_1\rangle - \mu\|\zeta_2\rangle \pm \gamma\|\zeta_4\rangle \mp \beta\|\zeta_3\rangle$
$\|00010\rangle + \|10011\rangle \pm \|01101\rangle \pm \|11100\rangle$	$\alpha\|\zeta_3\rangle + \mu\|\zeta_4\rangle \pm \gamma\|\zeta_6\rangle \pm \beta\|\zeta_5\rangle$
$\|00010\rangle - \|10011\rangle \pm \|01101\rangle \mp \|11100\rangle$	$\alpha\|\zeta_3\rangle - \mu\|\zeta_4\rangle \pm \gamma\|\zeta_6\rangle \mp \beta\|\zeta_5\rangle$
$\|00110\rangle + \|10111\rangle \pm \|01001\rangle \pm \|11000\rangle$	$\alpha\|\zeta_7\rangle + \mu\|\zeta_8\rangle \pm \gamma\|\zeta_2\rangle \pm \beta\|\zeta_1\rangle$
$\|00110\rangle - \|10111\rangle \pm \|01001\rangle \mp \|11000\rangle$	$\alpha\|\zeta_7\rangle - \mu\|\zeta_8\rangle \pm \gamma\|\zeta_2\rangle \mp \beta\|\zeta_1\rangle$
$\|00100\rangle + \|10101\rangle \pm \|01010\rangle \pm \|11011\rangle$	$\alpha\|\zeta_5\rangle + \mu\|\zeta_6\rangle \pm \gamma\|\zeta_3\rangle \mp \beta\|\zeta_4\rangle$
$\|00100\rangle - \|10101\rangle \pm \|01010\rangle \mp \|11011\rangle$	$\alpha\|\zeta_5\rangle - \mu\|\zeta_6\rangle \pm \gamma\|\zeta_3\rangle \mp \beta\|\zeta_4\rangle$

$$|\zeta_6\rangle = \frac{1}{\sqrt{2}}(-|0\rangle|\phi_+\rangle + |1\rangle|\psi_+\rangle), \tag{29}$$

$$|\zeta_7\rangle = \frac{1}{\sqrt{2}}(|0\rangle|\phi_-\rangle - |1\rangle|\psi_-\rangle), \tag{30}$$

$$|\zeta_8\rangle = \frac{1}{\sqrt{2}}(|0\rangle|\psi_+\rangle + |1\rangle|\phi_+\rangle). \tag{31}$$

Now, Bob can perform a measurement in the basis $|0\rangle, |1\rangle$ and convey the result of his measurement to Charlie after which Charlie can apply an appropriate unitary transformation on his qubits to get back $|\psi_b\rangle$. For instance, if the combined state of Bob and Charlie collapses to the third state given in table 4, then the outcome of the measurement performed by Bob and the state obtained by Charlie are shown in table 5.

Now Charlie can apply an appropriate unitary operator on his qubits and get back $|\psi_b\rangle$ as shown in table 6.

Table 5. The outcome of the measurement performed by Bob and the state obtained by Charlie.

Outcome of the measurement	State obtained
$\vert 0\rangle$	$\alpha\vert\psi_+\rangle + \mu\vert\phi_-\rangle \mp \gamma\vert\phi_+\rangle \mp \beta\vert\psi_-\rangle$
$\vert 1\rangle$	$-\alpha\vert\phi_+\rangle + \mu\vert\psi_-\rangle \pm \gamma\vert\psi_+\rangle \mp \beta\vert\phi_-\rangle$

Table 6. Set of unitary operators required to obtain $\vert\psi_b\rangle$.

State obtained	Unitary operator to be used
$\alpha\vert\psi_+\rangle + \mu\vert\phi_-\rangle \mp \gamma\vert\phi_+\rangle \mp \beta\vert\psi_-\rangle$	$\vert 00\rangle\langle\psi_+\vert + \vert 10\rangle\langle\phi_-\vert \mp \vert 01\rangle\langle\phi_+\vert \mp \vert 11\rangle\langle\psi_-\vert$
$-\alpha\vert\phi_+\rangle + \mu\vert\psi_-\rangle \pm \gamma\vert\psi_+\rangle \mp \beta\vert\phi_-\rangle$	$-\vert 00\rangle\langle\phi_+\vert + \vert 10\rangle\langle\psi_- \pm \vert 01\rangle\langle\psi_+\vert \mp \vert 11\rangle\langle\phi_-\vert$
$\alpha\vert\psi_+\rangle - \mu\vert\phi_-\rangle \mp \gamma\vert\phi_+\rangle \mp \beta\vert\psi_-\rangle$	$\vert 00\rangle\langle\psi_+\vert - \vert 10\rangle\langle\phi_-\vert \mp \vert 01\rangle\langle\phi_+\vert \mp \vert 11\rangle\langle\psi_-\vert$
$-\alpha\vert\phi_+\rangle - \mu\vert\psi_-\rangle \pm \gamma\vert\psi_+\rangle \pm \beta\vert\phi_-\rangle$	$-\vert 00\rangle\langle\phi_+\vert - \vert 10\rangle\langle\psi_- \pm \vert 01\rangle\langle\psi_+\vert \pm \vert 11\rangle\langle\phi_-\vert$
$\alpha\vert\phi_+\rangle + \mu\vert\psi_-\rangle \pm \gamma\vert\phi_-\rangle \pm \beta\vert\psi_+\rangle$	$\vert 00\rangle\langle\phi_+\vert + \vert 10\rangle\langle\psi_-\vert \pm \vert 01\rangle\langle\psi_-\vert \pm \vert 11\rangle\langle\phi_-\vert$
$\alpha\vert\psi_+\rangle - \mu\vert\phi_-\rangle \pm \gamma\vert\psi_-\rangle \mp \beta\vert\phi_+\rangle$	$-\vert 00\rangle\langle\psi_+\vert + \vert 10\rangle\langle\phi_- \pm \vert 01\rangle\langle\psi_-\vert \mp \vert 11\rangle\langle\phi_+\vert$
$\alpha\vert\phi_+\rangle - \mu\vert\psi_-\rangle \pm \gamma\vert\phi_-\rangle \mp \beta\vert\psi_+\rangle$	$\vert 00\rangle\langle\phi_+\vert + \vert 10\rangle\langle\psi_-\vert \pm \vert 01\rangle\langle\psi_-\vert \mp \vert 11\rangle\langle\phi_-\vert$
$\alpha\vert\psi_+\rangle - \mu\vert\phi_-\rangle \pm \gamma\vert\psi_-\rangle \mp \beta\vert\phi_+\rangle$	$-\vert 00\rangle\langle\psi_+\vert + \vert 10\rangle\langle\phi_- \pm \vert 01\rangle\langle\psi_-\vert \mp \vert 11\rangle\langle\phi_+\vert$
$\alpha\vert\phi_-\rangle + \mu\vert\psi_+\rangle \pm \gamma\vert\psi_-\rangle \pm \beta\vert\phi_+\rangle$	$\vert 00\rangle\langle\phi_-\vert + \vert 10\rangle\langle\psi_+\vert \pm \vert 01\rangle\langle\psi_-\vert \pm 11\rangle\langle\phi_+\vert$
$-\alpha\vert\psi_-\rangle + \mu\vert\phi_+\rangle \mp \gamma\vert\phi_-\rangle \pm \beta\vert\psi_+\rangle$	$-\vert 00\rangle\langle\psi_-\vert + \vert 10\rangle\langle\phi_+ \mp \vert 01\rangle\langle\phi_-\vert \pm \vert 11\rangle\langle\psi_+\vert$
$\alpha\vert\phi_-\rangle - \mu\vert\psi_+\rangle \pm \gamma\vert\psi_-\rangle \mp \beta\vert\phi_+\rangle$	$\vert 00\rangle\langle\phi_-\vert - \vert 10\rangle\langle\psi_+\vert \pm \vert 01\rangle\langle\psi_-\vert \mp \vert 11\rangle\langle\phi_+\vert$
$-\alpha\vert\psi_-\rangle - \mu\vert\phi_+\rangle \mp \gamma\vert\phi_-\rangle \mp \beta\vert\psi_+\rangle$	$-\vert 00\rangle\langle\psi_-\vert - \vert 10\rangle\langle\phi_+ \mp \Vert 01\rangle\langle\phi_-\vert \mp \Vert 11\rangle\langle\psi_+\vert$
$-\alpha\vert\psi_-\rangle - \mu\vert\phi_+\rangle \pm \gamma\vert\psi_+\rangle \pm \beta\vert\phi_-\rangle$	$-\vert 00\rangle\langle\psi_-\vert - \vert 10\rangle\langle\phi_+\vert \pm \vert 01\rangle\langle\psi_+\vert \pm \vert 11\rangle\langle\phi_-\vert$
$-\alpha\vert\phi_-\rangle + \mu\vert\psi_+\rangle \mp \gamma\vert\phi_+\rangle \pm \beta\vert\psi_-\rangle$	$-\vert 00\rangle\langle\phi_-\vert + \vert 10\rangle\langle\psi_+ \mp \vert 01\rangle\langle\phi_+\vert \pm \vert 11\rangle\langle\psi_-\vert$
$-\alpha\vert\psi_-\rangle + \mu\vert\phi_+\rangle \pm \gamma\vert\psi_+\rangle \mp \beta\vert\phi_-\rangle$	$-\vert 00\rangle\langle\psi_-\vert + \vert 10\rangle\langle\phi_+\vert \pm \vert 01\rangle\langle\psi_+\vert \mp \vert 11\rangle\langle\phi_-\vert$
$-\alpha\vert\phi_-\rangle - \mu\vert\psi_+\rangle \mp \gamma\vert\phi_+\rangle \mp \beta\vert\psi_-\rangle$	$-\vert 00\rangle\langle\phi_-\vert - \vert 10\rangle\langle\psi_+ \mp \vert 01\rangle\langle\phi_+\vert \mp \vert 11\rangle\langle\psi_-\vert$

4. Conclusion

In this paper, we have demonstrated the usefulness of a recently introduced six-qubit state for the teleportation of an arbitrary three-qubit state and for the quantum state sharing of an arbitrary two-qubit state in two distinct ways. Further, this state satisfies the conjecture made by two of the present authors [19] that the number of distinct ways in which one can split an arbitrary n-qubit state using a genuinely entangled N-qubit state as an entangled channel among two parties in the case where they need not meet up is $(N - 2n)$. The spectacular properties of this state make our protocols robust against decoherence [25]. In future, we wish to study these protocols through noisy channels and investigate the decoherence properties of this state.

Acknowledgments

The authors acknowledge Professor V Singh of Homi Bhabha Centre for Science Education (HBCSE, TIFR), Mumbai, India for continuous encouragement.

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